Eur. Phys. J. B **8**, 665–669 (1999)

THE EUROPEAN PHYSICAL JOURNAL B
EDP Sciences

Low q-moment multifractal analysis of Gold price, Dow Jones Industrial Average and BGL-USD exchange rate

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Received 17 October 1998 and Received in final form 2 November 1998

Abstract. An estimate of the low $q-$moment values of the assumed multifractal spectrum of Gold price, Dow Jones Industrial Average (DJIA) and Bulgarian Lev - USA Dollar (BGL-USD) exchange rate over a 6 1/2 year time span has been made. The findings can be compared to the analysis made on 23 foreign currency exchange rates by Vandewalle and Ausloos but there is a clear indication of some differences. Comparison to fractional Brownian motion is made. The analysis shows that these three financial data are not likely fractal but rather multifractal indeed.

PACS. 05.40.+j Fluctuation phenomena, random processes, and Brownian motion – 01.75.+m Science and society

1 Introduction

Evolution of foreign currency exchange rates are clearly not ordinary random walks [1,2]: temporal correlations exist. However, the nature of these correlations is actually not clear. Indeed, various models provide temporal evolutions quite similar to real currency walks [3].

It seems that a primary consideration to be discussed is the question of stationarity and also that of intermittency (or not) of the data. This question occurs when discussing the legitimacy of the Auto-Regressive Conditional Heteroscedasticity (ARCH) modelling [2,4,5]. It has been shown that covariance stationarity, which is a fundamental hypothesis of ARCH modelling, is implausible for daily data [5], although ARCH together with generalized econometric models like GARCH [6] remain the most popular models in the finance community. It can be shown from a statistical physics point of view that these models are still Brownian motion like [7] and thus fail to provide the needed analysis about persistence or not of an index evolution.

Recently, the interest of the statistical physics community [8–36] for economic time dependent data series, has been revived quite a relatively long time after pioneering papers like that of Mandelbrot in 1959 [37]. Indeed, statistical physicists are familiar with the self-organization of large systems containing many interacting agents [38], systems thus apparently similar to those encountered in the economic and financial fields.

A fundamentally and physically oriented question is the existence (or not) of long-range power-law correlations [3,7] and short range ones [36] in foreign exchange rates. Some time ago, the fractal nature has been discussed for such data, but the multifractal concepts [39] which seem more sensible or realistic has already been tested [40,41]. A multi-affine analysis [35,42] of several foreign currency exchange rates has been recently presented by Vandewalle and Ausloos [41]. Sometimes a generalization of the multifractal formalism to singular functions (signals) based on the wavelet analysis is prefered [13,33,34,43]. This method has been recently applied to financial data [15]. In the following we consider the low moments of the multifractal spectrum for some specific financial data, *i.e.* Dow Jones Industrial Average, Gold price and BGL-USD exchange rate (Fig. 1).

From now on the data are normalized to unity on the maximum value obtained during the examined time interval, *i.e.* between February 1991 and May 1997. These are daily data. Weekends and holidays have not been taken into account in order to get some apparent continuity (or identical intervals) along the ordinate scale. These were recently used in order to search for short range correlations through the low order variability diagrams [36]. The Dow Jones and Gold price evolutions are new types of indices examined with respect to the H_1, C_1 phase diagram introduced in [41] in financial data, but already used in turbulence and meteorology [44,45]. The BGL-USD exchange rate can serve as a test with respect to the 23 points reported in reference [41].

2 Theoretical framework

The technique consists in calculating the so-called "qth order height-height correlation function" or "qth order

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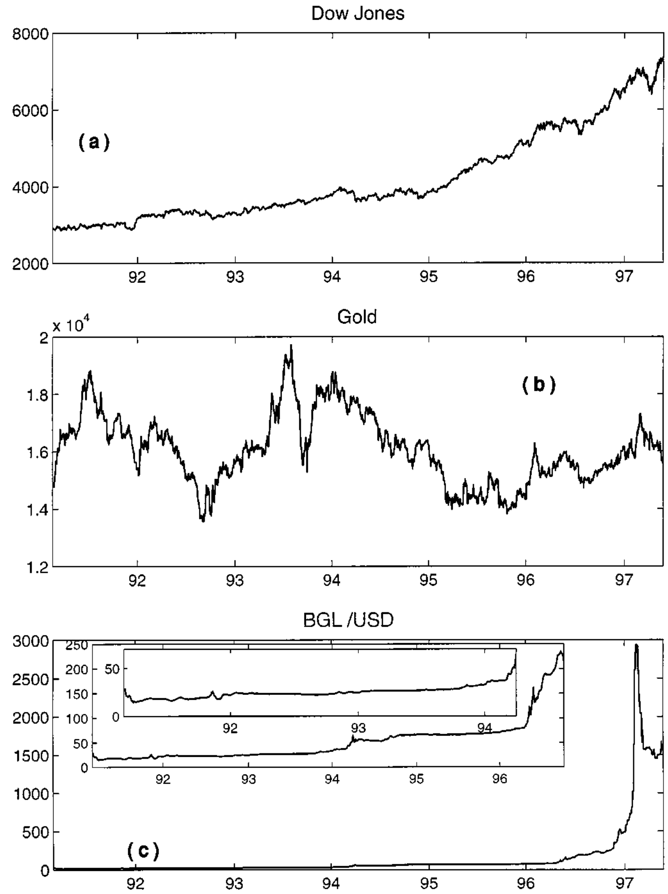

Fig. 1. (a) Dow Jones Industrial Average index (DJIA) time evolution. (b) Gold (GLD) price evolution between February 1991 and May 1997. (c) Exchange rate variability of Bulgarian currency (BGL) with respect to the USA dollar (USD).

structure function" [46] of the normalized time-dependent signal $y(t_i)$, $\quad i = 0, \ldots, \Lambda$

$$c_q(\tau) = \langle |y(t_{i+r}) - y(t_i)|^q \rangle_\tau \tag{1}$$

where only non-zero terms are considered in the average $\langle . \rangle_\tau$ taken over all couples (t_{i+r}, t_i) such that $\tau = |t_{i+r} - t_i|$.

The generalized Hurst exponent $H(q)$ is defined through the relation

$$c_q(\tau) \propto \tau^{qH(q)}, \quad q \geq 0. \tag{2}$$

The structure function analysis provide an estimate of the nonstationarity of the data. The intermittency of the signal is studied through the singular measure analysis of the small-scale gradient field obtained from the data through

$$\varepsilon(r;l) = \frac{r^{-1} \sum_{i=l}^{l+r-1} |y(t_{i+r}) - y(t_i)|}{\langle |y(t_{i+r}) - y(t_i)| \rangle} \tag{3}$$

with

$$i = 0, \ldots, \Lambda - r \tag{4}$$

and

$$r = 1, 2, \ldots, \Lambda = 2^m. \tag{5}$$

The scaling properties of the generating function are searched for through

$$\chi_q(\tau) = \langle \varepsilon(r;l)^q \rangle \sim \tau^{-K(q)}, \quad q \geq 0 \tag{6}$$

with τ as defined above. The $K(q)$-exponent is closely related to the generalized dimensions $D_q = 1 - K(q)/(q-1)$ [47]. The nonlinearity of both characteristics exponents, $qH(q)$ and $K(q)$, describes the multifractality of the signal. If a linear dependence is obtained, then the signal is monofractal or in other words, the data follows a simple scaling law for these values of q.

Among the exponents $H(q)$ and D_q, the *roughness* H_1 and the *sparseness* $C_1 = 1 - D_1$ have a physically meaningful interpretation [48]. The $q = 0, 1$, and 2 moments have some known physical relevance at this time through generalized fractal dimensions [39]. The other moments have a less obvious physical meaning.

3 Data analysis

Following the above lines of calculation, the results for the Gold ingot (GLD) price, Dow Jones Industrial Average (DJIA) and BGL-USD exchange rate, as normalized (see above) are given in Figures 2a to 2c. Statistical error bars are indicated when significative, *i.e.* for q-values greater than one in general. They are estimated following standard statistical data analysis [49]. The $qH(q)$ function is smoothly rising from zero such as the nonlinear dependence, and therefore, the multifractality of the series, are most pronounced for BGL-USD data. The $K(q)$ function appears, like in other cases [44,45] to have a small minimum below $q = 1$. For $q > 2$ it rises with a linear slope ~ 1.25 for the Dow Jones and the GLD series, but ~ 0.6 for the BGL-USD series. It can be seen that C_1 will be different for different indices, though C_1 is rather a local measure at $q = 1$. Moreover, if a multifractality exists, $K(q)$ should be nonlinear. It is rather monofractal when a linear behavior is obtained, but this might be debated as originating from sampling limitations.

For values of q above $q = 2$ the sample size becomes noticeable. The structure functions seem to reach an almost constant regime, while the singular measures tend towards a linear dependence. Also the larger error bars of the BGL-USD series are interpreted to be due to the higher sensitivity toward sampling limitations of the signal related to the higher extremas in different scales. Such properties of the characteristic exponents are somewhat expected because of the relatively short data series used here. However, a nontrivial dependence for small and intermediate values of q is observed. Nevertheless the concavity of the $qH(q)$-function can be interpreted as denoting the multifractality of the financial series. The convexity of the $K(q)$-function indicates the multi-affinity of the absolute gradients of the signals at small scales.

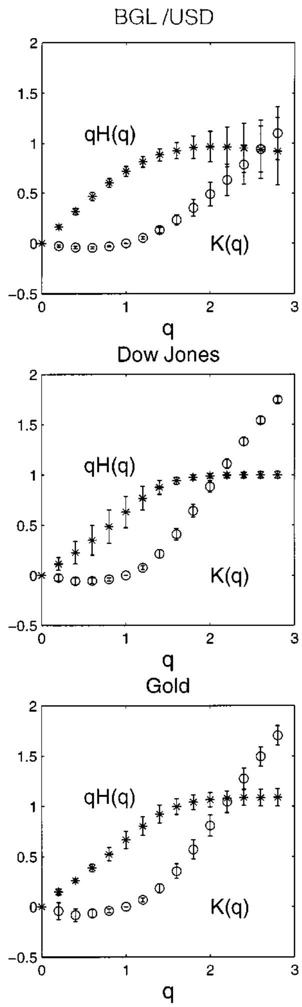

Fig. 2. The low q-dispersion of $qH(q)$ and $K(q)$ for (a) the exchange rate Bulgarian currency BGL to USD; (b) the Dow Jones Industrial average; (c) the Gold price. Error bars are indicated.

Because the time series are rather short it could be argued that intermittent and non-intermittent signals cannot be discriminated upon. Some discussion about the relevance of statistical errors on the $H(q)$ spectrum can be found in reference [24].

4 Discussion

The roughness (Hurst) exponent H_1 has been calculated from the correlation function $c_1(\tau)$ supposed to behave like

$$c_1(\tau) = \langle |y(t_{i+r}) - y(t_i)| \rangle_\tau \sim \tau^{H_1}. \quad (7)$$

The roughness exponent H_1 describes the excursion of the signal. For the classical random walk (Brownian motion) [51], $H_1 = 1/2$; for a *persistent* signal, $H_1 > 1/2$; and for an *anti-persistent* signal, $H_1 < 1/2$. The C_1 exponent [46,48] is a measure of the intermittency lying in the signal $y(t)$

$$C_1 = \left. \frac{dK(q)}{dq} \right|_{q=1} \quad (8)$$

which can be numerically estimated by measuring $K(q)$ around $q = 1$.

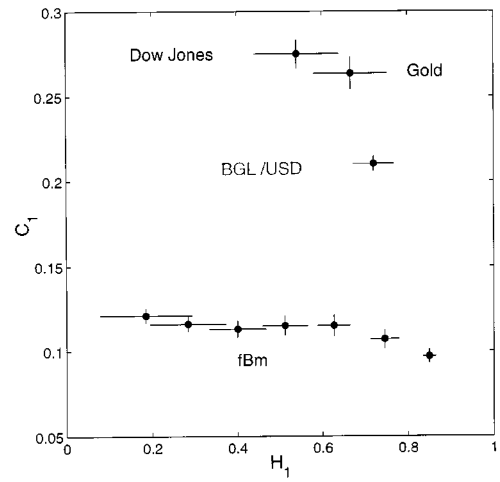

Fig. 3. The (H_1,C_1) phase diagram with data points for three financial indices and fractional Brownian motion (fBm) as studied in the text.

Table 1.

	H_1	C_1
Dow Jones	0.5408±0.0999	0.213±0.011
Gold	0.6675±0.0859	0.196±0.012
BGL-USD	0.7208±0.0481	0.209± 0.046
fBm		
$H = 0.2$	0.1859±0.1080	0.121±0.0042
$H = 0.3$	0.2846±0.0895	0.116±0.0045
$H = 0.4$	0.4010±0.0670	0.113±0.0050
$H = 0.5$	0.5124±0.0537	0.115±0.0058
$H = 0.6$	0.6270±0.0384	0.115±0.0063
$H = 0.7$	0.7459±0.0338	0.107±0.0055
$H = 0.8$	0.8495±0.0157	0.097±0.0041

The values of the *roughness* H_1 and the spareness or *intermittency* parameter C_1 as obtained from Figure 3 are given in Table 1.

From the above BGL-USD data it can be observed that $H_1 = 0.72$, larger than the value found for other financial exchange rates by Vandewalle and Ausloos [41]. Moreover the values of the GLD and DJIA variations fall nicely in the (H_1, C_1) diagram (Fig. 3) in the vicinity of the values corresponding to other foreign exchange data and to microwave signals of the water vapor distribution in the atmosphere for which H_1 can be between 0.61 and 0.68 [45], and quite away from turbulence field data exponents ($H_1 \sim 0.33$). It was also shown in [41] that time series generated by the ARCH models have low intermittency and look like a simple random walk from this (H_1, C_1) point of view.

It should be hereby emphasized that the intermittent contribution ($C_1 \neq 0$) suggests that rare and large jumps take inherently place in some data excursions. We found rather large intermittency $C_1 \approx 0.2$ for the three financial data of interest here in contrast to the exchange market intermittency reported by other authors [35,41]. The density of these "jumps" is to be compared to that found in geophysical signals like the evolution of the liquid content in clouds for which $C_1 \approx 0.10$ [48], and microwave signals from atmospheric water vapor content, where C_1 varies between 0.10 and 0.15 [45].

Of course both H_1 and C_1 values may change with time and depend on the interval over which the numerical analysis is performed. This is certainly one major reason among others to consider a multifractal analysis and spectrum.

5 Fractional Brownian motion

Precise reference signals are required to evaluate methods for characterizing a fractal time series and before reaching any definite conclusion [50]. In order to test the reliability of the above analysis on one hand, and to attempt to provide a non trivial set of models for this type of financial data behavior we have used the above low $q-$moment multifractal analysis method on fractional (univariate) Brownian motion (fBm) [51,52]. The number of data points has been taken identical to that of the financial data series. The examined fBm series have the predefined Hurst exponent $H_1 = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$.

The resulting $qH(q)$ and $K(q)$ functions for selected fBm curves ($H_1 = 0.3, 0.5, 0.8$) are given in Figure 4, while the corresponding (H_1, C_1) values are reported in Figure 3. As it was expected for all fBm cases the $qH(q)$ dependence of q is a straight line with slope H_1. Table 1 contains (H_1, C_1) values for all fBm cases. Moreover the relationship between the predefined and numerically obtained values of H_1 for the fBm is given in Figure 5. It is observed that there is no systematic deviation of the latter values. In contrast the values of C_1 for the fBm signals are of the order of 0.1 and slowly varying even within the error bars. Since $C_1 = 1 - D_1$, where D_1 is the information dimension [47,53], it appears that such a finite value of C_1 is likely due to the finite size of the series. The relevance of finite size effects in fractal analysis is well-known and indicates that the signals are only *quasi-fractal* indeed. This should be taken into account when comparing data analyzed from different "samples". However the above analysis allows us to have some insight on extrinsic error bars in contrast to intrinsic ones due to the data *a priori* assumed statistical distribution itself, *i.e.* a Gaussian one, as those indicated in the figures.

Finally, even though the low $q > 0$ region of a fractal signal can be easily treated, one should be aware that difficulties arise in applying a multifractal analysis for the $q < 0$ region to the height-height correlation function [8,33], and therefore a full multifractal analysis is very arduous. The more so here because the relatively small statistics and the interplay between intrinsic and extrinsic error bars in most financial data do not seem to be well understood at this time. Tests of various physics-like models should thus request also some further thought. Nevertheless, we have shown that a significant intermittent component ($C_1 \neq 0$) exists and depends on the nature of the trading market.

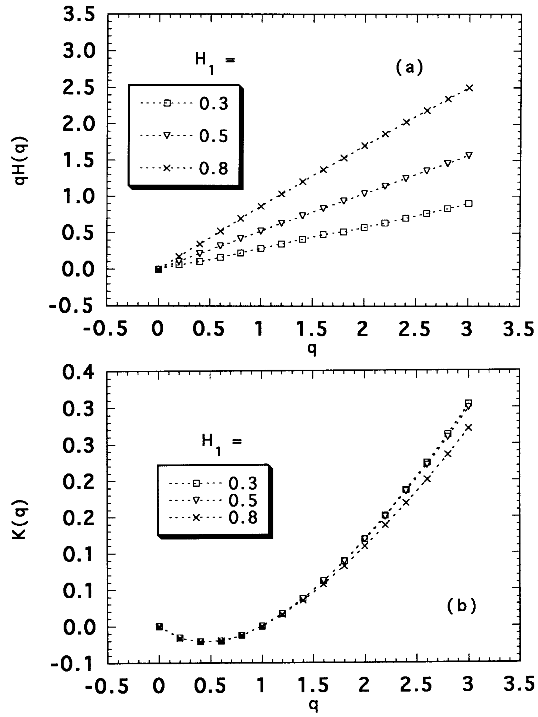

Fig. 4. The low q-dispersion $qH(q)$ and $K(q)$ functions of selected fBm time series.

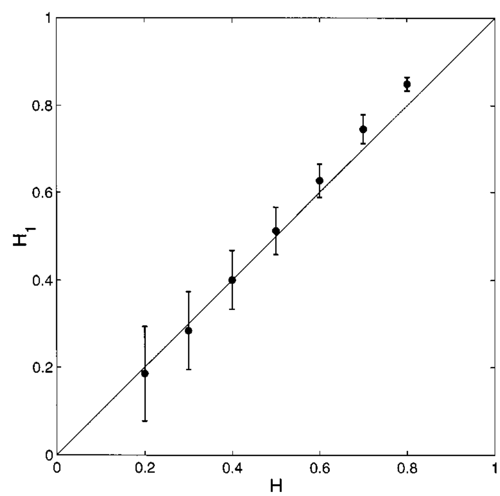

Fig. 5. The measured H_1 *vs.* the predetermined Hurst exponent value of a few fBm time series.

This work was partially supported through the ARC (94-99/174) contract of the University of Liège. Commissariat General aux Relations Internationales (CGRI)-Academy of Sciences of Bulgaria agreements allowed some stay of KI in

Belgium and MA in Bulgaria when parts of this work were performed. Discussions with N. Vandewalle are gratefully acknowledged.

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